properties of the igniter and condensed medium. A similar analysis may be carried out on a more general assumption as regards parameter γ. If $\gamma = \gamma_0 \alpha(\beta)$ with $\gamma_0 = O(1)$, the igniter temperature variation is substantial for $\sigma(\beta)\alpha^{-2}(\beta)\beta^{-3} = O(1)$. In that case the variables in the inner zone are to be of the form $x = (r-1)\alpha\beta^2$ and $\tau = t\alpha^2\beta^2$, and for α satisfying the inequality $\alpha\beta^2 \gg 1$, the solution is determined by the derived here formulas in which $\delta / \alpha^2\beta^3$ and γ / α are to be substituted for δ_0 and γ, respectively. If, however, $\alpha = \beta^{-2}$ the problem reduces to the solution of an equation in which the differential operator retains the form determined by the problem symmetry.

We note in conclusion that the problem of igniting a reacting gas by a heated body with allowance for the cooling of the igniter and the burnout of reagent can be treated by the method developed here. In that case the problem reduces to the integration of two nonlinear integral equations for the igniter temperature and concentration of reagent at its surface.

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THE REPRESENTATION OF THE DISPLACEMENT GRADIENT OF ISOTROPIC ELASTIC BODY IN TERMS OF THE PIOLA STRESS TENSOR

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The representation of the displacement gradient of an isotropic elastic body is analyzed. It is shown on the basis of a single controlling inequality and a polar expansion of the Piola tensor that such representation has generally four branches. The mechanical meaning and the nature of that ambiguity is explained. It is established that when the angles of turn of material fibers are not excessively large, only one of the four branches is obtained. Particular cases in which the nature of ambiguity is more complex are investigated. It is noted that in many practical problems the representation of the displacement gradient by the Piola stress tensor is unambiguous.

The considered problem is associated with the variational principle of complementary energy in the nonlinear theory of elasticity, where the statistically feasible fields of the asymmetric Piola stress tensor is varied [1]. A method was proposed there for expressing the displacement gradient in terms of the Piola stress tensor for an isotropic elastic body. Later the concept of complementary energy and the representation of the strain gradient in terms of the Piola stress tensor were considered in [2, 3]. Examples of the use of the complementary energy concept are given in [2] and the case of an anisotropic body is considered in [3]. These investigations disclosed that the considered representation of the strain tensor leads to ambiguity, but the character and nature of the ambiguity were not fully investigated.

1. The following notation is used for tensor quantities that define the stress-strain state of a continuous medium [4]: C for the tensor-gradient of the radius vector of the deformed body point, which is also called strain gradient, $U = (C \cdot C^T)^{1/2}$ for the positive definite square root of the metric of the Cauchy strain, $A = U^{-1} \cdot C$ for the tensor of turn of the principal axes of deformation, D for the asymmetric Piola stress tensor, and $Q = D \cdot C^{-1}$ for the symmetric stress energy tensor. When strains are defined relative to the undeformed state, tensor Q for an isotropic elastic body is an isotropic function of tensor U and, consequently, the principal axes of Q and U coincide. Hence tensor S defined by formula

$$S = D \cdot A^T = Q \cdot U \tag{1.1}$$

for an isotropic material is symmetric, and also an isotropic function of U

$$S = S(U) \tag{1.2}$$

If the inverse function of (1.2) $U = U(S)$ is known, the problem of representing the strain gradient C in terms of tensor D reduces to the determination of the expression $A(D)$ of the turn tensor in terms of the Piola stress tensor. In fact, from (1.1) we have

$$C = U \cdot A = U(S) \cdot A = U[D \cdot A^T(D)] \cdot A(D) \tag{1.3}$$

The single-valued solvability of Eq. (1.2) for tensor U can be proved if one takes into consideration that the material conforms to the controlling GCN-inequality [5], which imposes on the response function $D(C)$ of an elastic material the following restraints:

$$[D(C') - D(C)] \cdot\cdot (C'^T - C^T) > 0, \quad C' = C \cdot P \tag{1.4}$$

for any nonsingular tensor C or any symmetric positive definite tensor different from the unit tensor P. Inequality (1.4) can be presented as

$$[Q(C') \cdot C' - Q(C) \cdot C] \cdot\cdot (C'^T - C^T) > 0 \tag{1.5}$$

If we take any arbitrary symmetric positive definite tensor P' coaxial with P as tensor C, we obtain

$$C = U = P', \quad C' = U' = P' \cdot P \tag{1.6}$$

For any arbitrary coaxial positive definite and not coinciding U' and U from (1.5) and (1.6) we have

$$[Q(U') \cdot U' - Q(U) \cdot U] \cdot\cdot (U' - U) > 0 \tag{1.7}$$

or in conformity with (1.1)

$$[S(U') - S(U)] \cdot\cdot (U' - U) > 0 \tag{1.8}$$

Taking into account the coaxiality of tensors S and U and representing these on the basis of principal directions, instead of (1.8) we obtain

$$\sum_{n=1}^{3} [S_n(U_1', U_2', U_3') - S_n(U_1, U_2, U_3)](U_n' - U_n) > 0 \qquad (1.9)$$

$$\mathrm{S} = \sum_{n=1}^{3} S_n \mathbf{e}_n \mathbf{e}_n, \quad \mathrm{U} = \sum_{n=1}^{3} U_n \mathbf{e}_n \mathbf{e}_n$$

Inequality (1.9) shows that when $U_n' \neq U_n$ for at least one n, then $S_m' \neq S_m$ for at least one m, which means that different values of U correspond to different S, hence U (S) is a single-valued function.

Let us pass to the determination of the dependence A (D) for an isotropic material. The problem reduces to finding the tensor of turn using the equation

$$\mathrm{D} \cdot \mathrm{A}^T = \mathrm{A} \cdot \mathrm{D}^T \qquad (1.10)$$

which defines the property of symmetry of tensor S.

Since the deformation of a continuous medium establishes a one-to-one correspondence between the coordinates of points of the deformed and undeformed body, the strain gradient C is a nonsingular tensor, and by suitable selection of the system of coordinates it is always possible to have a positive det C.

It follows from the theorem of polar expansion that det A $= 1$, i.e. A is a properly orthogonal tensor. Hence only properly orthogonal solutions of Eq. (1.10) have a physical meaning.

It is shown in [6] that any tensor of second rank can be represented in the form of the product of a symmetric nonnegative tensor by some orthogonal tensor. For the Piola stress tensor we have

$$\mathrm{D} = \mathrm{K} \cdot \mathrm{N} \qquad (1.11)$$

where N is an orthogonal tensor and K the nonnegative square root of tensor $\mathrm{D} \cdot \mathrm{D}^T$ uniquely determined by the specified tensor D. From (1.10) and (1.11) we obtain

$$\mathrm{K} \cdot \mathrm{H} = \mathrm{H}^T \cdot \mathrm{K} \qquad (1.12)$$

where the orthogonal tensor H is determined by

$$\mathrm{H} = \mathrm{N} \cdot \mathrm{A}^T \qquad (1.13)$$

Further analysis depends on the properties of tensor $\mathrm{D} \cdot \mathrm{D}^T$ eigenvalues.

2. First, let us consider the case in which tensor D and, consequently, also K are nonsingular. All eigenvalues of K are then positive and the orthogonal tensor N is uniquely determined by formula

$$\mathrm{N} = \mathrm{K}^{-1} \cdot \mathrm{D} \qquad (2.1)$$

From Eq. (1.12) we have

$$(\mathrm{K} \cdot \mathrm{H})^2 = \mathrm{K}^2 \quad \text{or} \quad \mathrm{K} \cdot \mathrm{H} = \sqrt{\mathrm{K}^2} = \sqrt{\mathrm{D} \cdot \mathrm{D}^T} \qquad (2.2)$$

According to (1.12) only symmetric square roots of tensor K^2 are of interest. The general expression for these is of the form

$$\mathrm{K} \cdot \mathrm{H} = \pm k_1 \mathbf{e}_1 \mathbf{e}_1 \pm k_2 \mathbf{e}_2 \mathbf{e}_2 \pm k_3 \mathbf{e}_3 \mathbf{e}_3 \qquad (2.3)$$

where $\mathbf{e}_s$ are unit vectors of principal axes of tensor K and k_s are eigenvalues of K. Since tensor K is nonsingular, from (2.3) we obtain

$$\mathrm{H} = \pm \mathbf{e}_1\mathbf{e}_1 \pm \mathbf{e}_2\mathbf{e}_2 \pm \mathbf{e}_3\mathbf{e}_3 = \mathrm{H}^T \tag{2.4}$$

When the eigenvalues of K are simple, the diades $\mathbf{e}_s\mathbf{e}_s$ (do not sum by s) are uniquely defined, and there are eight solutions for the orthogonal tensor H. From (2.4), (1.13) and (2.1) we have

$$\mathrm{A} = \mathrm{H}\cdot\mathrm{K}^{-1}\cdot\mathrm{D} \tag{2.5}$$

In accordance with (2.4) four values of tensor H have their determinants equal unity, while for the remaining four $\det \mathrm{H} = -1$. Hence only four solutions satisfy the condition $\det \mathrm{A} = 1$, The related tensors H must be selected so as to have $\det \mathrm{H}$ and $\det \mathrm{D}$ of the same sign.

Thus in the most general case of simple nonzero eigenvalues of tensor $\mathrm{D}\cdot\mathrm{D}^T$ the representation of the tensor of turn in terms of the Piola stress tensor has four branches which differ from each other by a 180° turn about each principal axis of tensor K. This ambiguity is of a fundamental character and has a mechanical meaning.

Let us imagine a parallelepiped of elastic material subjected to an arbitrary affine deformation. It can be seen on the basis of the physical meaning of the Piola stress tensor [4] that by specifying tensor D we specify the external dead forces that are evenly distributed along the parallelepiped faces. The ambiguity of deformation is determined by the nature of dead forces which do not alter their direction in space. This is shown in

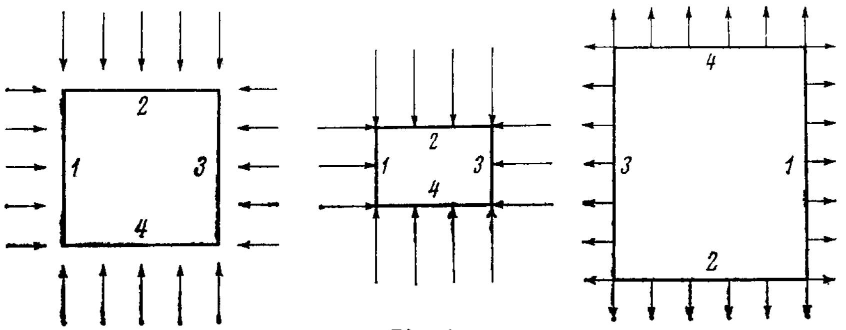

Fig. 1

Fig. 1 on the example of a rectangular parallelepiped subjected to normal forces. The undeformed body state (before application of forces) is shown there on the left and the two possible equilibrium states appear at the center and on the right of the diagram. In each of the equilibrium states, whose existence is evident, the forces, for instance, at face 1 have the same direction.

A particular case of the described here ambiguity was noted in [5] under different conditions as an instance of the ambiguity of equilibrium of an elastic body subjected to external dead forces.

Let A_1 be some properly orthogonal solution of Eq. (1.10). It can always be represented in the form

$$\mathrm{A}_1 = \mathbf{e}_1\mathbf{e}_1' + \mathbf{e}_2\mathbf{e}_2' + \mathbf{e}_3\mathbf{e}_3' \tag{2.6}$$

where $\mathbf{e}_s'$ is some orthonormalized basis. As shown above, there exist, besides this solution, solutions of the form

$$\mathrm{A}_r = \mathrm{H}_r\cdot\mathrm{A}_1 \quad (r = 1, 2, 3, 4) \tag{2.7}$$

$$\mathrm{H}_1 = \mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2 + \mathbf{e}_3\mathbf{e}_3, \quad \mathrm{H}_2 = \mathbf{e}_1\mathbf{e}_1 - \mathbf{e}_2\mathbf{e}_2 - \mathbf{e}_3\mathbf{e}_3 \tag{2.8}$$

$$\mathrm{H}_3 = -\mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2 - \mathbf{e}_3\mathbf{e}_3, \quad \mathrm{H}_4 = -\mathbf{e}_1\mathbf{e}_1 - \mathbf{e}_2\mathbf{e}_2 + \mathbf{e}_3\mathbf{e}_3$$

The following statement is valid. If one of the proper orthogonal tensors of the form (2.7) satisfies the inequality

$$\mathrm{tr}A > {}^5/_3 \tag{2.9}$$

then the remaining three do not satisfy it.

Let us assume that $\mathrm{tr}\, A_1 > {}^5/_3$. In accordance with (2.6) we have

$$e_1 \cdot e_1' + e_2 \cdot e_2' + e_3 \cdot e_3' > {}^5/_3 \tag{2.10}$$

It is obvious that

$$e_1 \cdot e_1' + e_2 \cdot e_2' + e_3 \cdot e_3' \leqslant 3 \tag{2.11}$$

since $e_1 \cdot e_1' \leqslant 1$, it follows from (2.10) that

$$-2\,(e_2 \cdot e_2' + e_3 \cdot e_3') < -{}^4/_3 \tag{2.12}$$

Adding inequalities (2.11) and (2.12) we obtain

$$e_1 \cdot e_1' - e_2 \cdot e_2' - e_3 \cdot e_3' < {}^5/_3 \tag{2.13}$$

which means that tensor A_2 does not satisfy condition (2.9). The proof for A_3 and A_4 is analogous.

The constant ${}^5/_3$ in inequality (2.6) is evidently the best, and cannot be decreased.

Let us use the representation of the proper orthogonal tensor in terms of the vector of finite turn [4]

$$A = (E - \mathbf{kk}) \cos \varphi + \mathbf{kk} - \mathbf{k} \times E \sin \varphi \tag{2.14}$$

where $\mathbf{k}$ is a unit vector which defines the direction of the axis of turn and φ is the angle of turn. From (2.14) we have

$$2 \cos \varphi = \mathrm{tr}\, A - 1$$

We see now that inequality (2.9) reduces to the following: $\cos \varphi > {}^1/_3$, hence

$$|\varphi| < \varphi^*, \quad \varphi^* \approx 70^\circ \tag{2.15}$$

Thus the geometrical meaning of inequality (2.9) that the finite angle of turn of the trihedron of the principal strain axes does not exceed 70°. Hence, if it is known that the angles of turn of material fibres are not excessive under deformation, the representation of the strain gradient in terms of the Piola stress tensor with allowance for (2.9) is unambiguous, since that inequality separates a single solution out of four possible.

Let us pass now to the case of multiple nonzero eigenvalues of K. If the eigenvalue is tripple, i.e. $D \cdot D^T$ is a spherical tensor, any orthonormalized basis can be taken as vector e_s in formula (2.4). In other words, in this case tensor H is an arbitrary symmetric orthogonal tensor. The general expression for such tensor with positive or negative determinant is of the form

$$H = \pm (E - \mathbf{ee}) + \mathbf{ee}\ \mathrm{sign}\ (\det D) \tag{2.16}$$

where E is the unit tensor and $\mathbf{e}$ an arbitrary unit vector. The sought tensor A is determined by formula (2.5).

Thus, when tensor K is spherical, then tensor of turn A is determined by Eq. (1.10) only to within a 180° turn about any axis.

In the case of double eigenvalue $k_1 = k_2$ the diade e_3e_3 is uniquely defined, and any arbitrary orthonormalized basis in the plane normal to e_3 may be taken as vectors e_1 and e_2. Vector $\mathbf{e}$ in formulas (2.16) is now not entirely arbitrary. It can be equal e_3 or $e_3 \times \mathbf{h} / \sqrt{1 - (e_3 \cdot \mathbf{h})^2}$, where $\mathbf{h}$ is any arbitrary unit vector different from e_3.

Thus, in the case of multiple roots of the characteristic equation of tensor $D \cdot D^T$ the ambiguity of representation A (D) is of a continual character, since the expression A(D) contains indeterminate parameters. The physical interpretation given in the case of simple roots is applicable in this case.

3. Let us consider the solution of Eq. (1.10) when $\det D = 0$, and assume that the spectrum of tensor K consists of one zero number $k_3 = 0$ and two different nonzero k_1 and k_2. The expression for tensor K is of the form

$$K = k_1 e_1 e_1 + k_2 e_2 e_2 \tag{3.1}$$

with the diades $e_1 e_1$ and $e_2 e_2$ uniquely determined.

Formula (2.1) is now inapplicable and the orthogonal co-factor N in the polar expansion of tensor D is nonunique. It is not difficult to see that its general expression is

$$N = e_1 e_1' + e_2 e_2' \pm e_3 e_3' \tag{3.2}$$

where the mutually orthogonal unit vectors e_α' $(\alpha = 1, 2)$ are determined by formulas (do not sum by $\alpha = 1, 2$)

$$e_\alpha' = k_\alpha^{-1} e_\alpha \cdot D \tag{3.3}$$

and vectors e_3 and e_3' are defined as follows:

$$e_3 = e_1 \times e_2, \quad e_3' = e_1' \times e_2' \tag{3.4}$$

According to (2.2) $K \cdot H = \pm k_1 e_1 e_1 \pm k_2 e_2 e_2$, hence $e_1 \cdot H = \pm e_1$ and $e_2 \cdot H = \pm e_2$. Because of the orthogonality of H we conclude that $e_3 \cdot H = \pm e_3$, which shows that in this case H is also of the form (2.4). From this with the use of (1.13) and (3.2) we obtain

$$A = \pm e_1 e_1' \pm e_2 e_2' \pm e_3 e_3' \tag{3.5}$$

Since by (3.4) the orthonormalized bases e_s and e_s' $(s = 1,2,3)$ are identically oriented, the proper orthogonal tensors in the set (3.5) are defined by

$$\begin{aligned} A &= \pm (k_1^{-1} e_1 e_1 + k_2^{-1} e_2 e_2) \cdot D + e_3 e_3' \\ A &= \pm (k_1^{-1} e_1 e_1 - k_2^{-1} e_2 e_2) \cdot D - e_3 e_3' \end{aligned} \tag{3.6}$$

A direct test will show the validity of the following identities:

$$\pm (k_1^{-1} e_1 e_2 + k_2^{-1} e_2 e_2) = \frac{g (I_1 + j_2) - D \cdot D^T}{\pm j_2 \sqrt{2 j_2 + I_1}}, \qquad j_2 = \sqrt{I_2} \tag{3.7}$$

$$\pm (k_1^{-1} e_1 e_1 - k_2^{-1} e_2 e_2) = \frac{g (I_1 + j_2) - D \cdot D^T}{\pm j_2 \sqrt{2 j_2 + I_1}}, \qquad j_2 = -\sqrt{I_2} \tag{3.8}$$

$$g = e_1 e_1 + e_2 e_2, \quad I_1 = k_1^2 + k_2^2 = \operatorname{tr}(D \cdot D^T)$$

$$I_2 = k_1^2 k_2^2 = {}^1/_2 \, [\operatorname{tr}^2 (D \cdot D^T) - \operatorname{tr} (D \cdot D^T)^2]$$

where I_1 and I_2 are, respectively, the first and second invariants of tensor $D \cdot D^T$. The equality (3.8) is to be understood in the sense that each of the two tensors in the left-hand side is equal to any tensor in the right-hand side. Instead of (3.6) we now have

$$\begin{aligned} A &= \frac{D (I_1 + j_2) - D \cdot D^T \cdot D}{\pm j_2 \sqrt{2 j_2 + I_1}} + e_3 e_3', \quad j_2 = \sqrt{I_2} \\ A &= \frac{D (I_1 + j_2) D \cdot D^T \cdot D}{\pm j_2 \sqrt{2 j_2 + I_1}} - e_3 e_3', \quad j_2 = -\sqrt{I_2} \end{aligned} \tag{3.9}$$

where it is taken into account in conformity with (1.11) and (3.1) that $g \cdot D = D$.

Introducing in the analysis vectors $f^\alpha = e_\alpha \cdot D = k_\alpha e_\alpha'$ (do not sum by $\alpha = 1, 2$), from (1.11), (3.1) and (3.2) we obtain

$$D = e_\alpha f^\alpha \tag{3.10}$$

On the basis of (3.4) we have

$$\sqrt{I_2}\, e_3 e_3' = (e_1 \times e_2)(f^1 \times f^2) = {}^1/_2 e_\alpha \times e_\beta f^\alpha \times f^\beta \qquad (\alpha, \beta = 1, 2) \tag{3.11}$$

Using the known formula for Levi-Civita symbols

$$e^{mns} e_{pqr} = \begin{vmatrix} \delta_p^m & \delta_q^m & \delta_r^m \\ \delta_p^n & \delta_q^n & \delta_r^n \\ \delta_p^s & \delta_q^s & \delta_r^s \end{vmatrix}$$

we can verify that the following identity

$$\mathbf{a} \times \mathbf{bx} \times \mathbf{y} = (\mathbf{a}\cdot\mathbf{xb}\cdot\mathbf{y} - \mathbf{a}\cdot\mathbf{yb}\cdot\mathbf{x})\, E - \mathbf{axyb} + \mathbf{b}\cdot\mathbf{xya} + \mathbf{a}\cdot\mathbf{yxb} - \mathbf{b}\cdot\mathbf{yxa} \tag{3.12}$$

is valid for any of vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{x}$ and $\mathbf{y}$.

Applying (3.12) to (3.11) we obtain

$$\sqrt{I_2}\, e_3 e_3' = {}^1/_2 (e_\alpha \cdot f^\alpha e_\beta \cdot f^\beta - e_\alpha \cdot f^\beta e_\beta \cdot f^\alpha)\, E + e_\beta \cdot f^\alpha f^\beta e_\alpha - e_\alpha \cdot f^\alpha f^\beta e_\beta$$

Note that according to (3.10)

$$f^\alpha e_\alpha = D^T, \qquad e_\beta \cdot f^\beta = \operatorname{tr} D^T = \operatorname{tr} D$$

$$e_\beta \cdot f^\alpha f^\beta e_\alpha = f^\beta e_\beta \cdot f^\alpha e_\alpha = (D^T)^2$$

We obtain the formula

$$(D^T)^2 - D^T \operatorname{tr} D^T + {}^1/_2 E\, [\operatorname{tr}^2 D^T - \operatorname{tr}(D^T)^2] \equiv D^{*T} \tag{3.13}$$

where D^* is the adjoint tensor of D. The matrix of mixed components of D^* in any arbitrary basis is the same as the transposed matrix of signed minors of matrix of components of D. For the nonsingular tensor X the equality $X^* = \det X X^{-1}$ is valid.

From (3.9) and (3.13) we finally obtain the following explicit form of the general solution of Eq. (1.10):

$$A = j_2^{-1} \{D^{*T} \pm (2j_2 + I_1)^{-1/2} [(I_1 + j_2) D - D\cdot D^T \cdot D]\}, \quad j_2 = \pm\sqrt{I_2} \tag{3.14}$$

We remind that this solution is valid under conditions

$$k_3 = 0,\ k_1 \neq 0,\ k_2 \neq 0,\ k_1 \neq k_2$$

or in the equivalent form

$$\det D = 0, \quad I_2 \neq 0, \quad I_1 \neq 2\sqrt{I_2}$$

In the case considered, as in that of the nonsingular tensor D, the expression for the tensor of turn has four branches that differ from each other by a 180° turn about each principal axis of tensor $D \cdot D^T$. The inequality (2.9) which separates from the four solutions the unique one is evidently applicable also in this case.

Note that the case of $\det D = 0$ in spite of being degenerate is fairly important, since it obtains in thin-walled structures where the Cauchy stress tensor may be often considered as two-dimensional (as, e.g., in the zero-moment theory of shells).

If two nonzero eigenvalues are the same ($k_1 = k_2$), only the principal axis of tensor

K which corresponds to the eigenvalue is uniquely determined, and any orthonormalized basis in a plane normal to e_3 can be taken as e_1 and e_2. In that case the tensor of turn is determined by the Piola stress tensor to within a 180° turn about e_3 and any axis normal to e_3.

It can be shown that, when only one eigenvalue is nonzero, $(K = k_1 e_1 e_1)$, the general solution of Eq. (1.10) contains the following indefiniteness: the turn by any arbitrary angle about vector e and a 180° turn about any axis normal to e_1.

Finally, when all three principal values of tensor K are zero, then $D = 0$ and any proper orthogonal tensor satisfies Eq. (1.10).

It has been, thus, established that in the case of simple roots of the characteristic equation of tensor $D \cdot D^T$, irrespective of whether tensor D is invertible, the representation C(D) has four separate branches. If it is *a priori* known (as it is in many practical problems) that the angles of turn of material fibres are not excessive, that representation in conjunction with inequality (2.9) is single-valued. In the case of multiple roots representation C (D) contains indeterminate parameters. However such cases are exceptional and in concrete problems they can appear only at certain surfaces or lines inside the region occupied by the body (i.e. on zero-measure sets). Owing to this, the indicated indefiniteness can be eliminated by passing to limit. Thus the representation of the dislocation gradient in terms of the Piola stress tensor is in many practical problems single-valued at each point of the body, and this makes possible the application of the variational principle of complementary energy for solving problems of the nonlinear theory of elasticity.

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